Intrinsic toroidal rotation in the scrape-off-layer

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1. Motivation and Summary

- Tokamak plasmas **spontaneously rotate** toroidally even in the absence of momentum injection.
- Intrinsic rotation is **important for ITER** where deposition of momentum will have a limited effect.
- Toroidal rotation can **stabilize** MHD instabilities and **reduce** turbulent transport.
- Experimental evidence for the **role of SOL flows** in determining core rotation profiles in L-mode [1].
- SOL flows can **determine the L-H power treshold** [2].

- A **simple theory** for intrinsic toroidal rotation in the SOL is presented here.
- **Results indicate** that
  - The sheath and the presence of pressure poloidal asymmetries act as sources of momentum
  - Momentum is transported radially by ballooning-like turbulent transport
- Global **3D simulations** with the analytical predictions.
- The analytical trends agree with main **observed experimental trends**.

2. SOL rotation theory

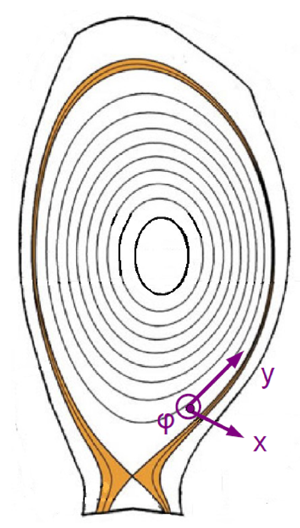
MODEL

- Within the drift-reduced Braginskii model [3]:

$$\frac{\partial v_{||i}}{\partial t} + v_{||i}\nabla_{||}v_{||i} + (\mathbf{v}_E \cdot \nabla)v_{||i} + \frac{1}{m_i n}\nabla_{||}p = 0$$

- Time-averaging:

$$\bar{v}_{||i}\nabla_{||}\bar{v}_{||i} + \frac{1}{B_\varphi}\langle\nabla\cdot\Gamma_v\rangle_t + \frac{1}{m_i\bar{n}}\nabla_{||}\bar{p} = 0$$



- $\langle\Gamma_{v,y}\rangle_t \simeq \Gamma_{v,y}^{EQ} = \sigma_\varphi \bar{v}_{||i}\frac{\partial\bar{\phi}}{\partial x}$
- $\langle\Gamma_{v,x}\rangle_t \simeq \langle\Gamma_{v,x}^{TURB}\rangle_t = -\sigma_\varphi\langle\bar{v}_{||i}\frac{\partial\bar{\phi}}{\partial y}\rangle_t$
- $\mathbf{B} = |B_\varphi|\,(\sigma_\varphi\hat{\mathbf{e}}_\varphi + \alpha\sigma_y\hat{\mathbf{e}}_y)$, $\alpha = |B_p|/|B_\varphi|$

ESTIMATE OF TURBULENT FLUX

- Linearising the parallel ion momentum equation:

$$\gamma\tilde{v}_{||i} \simeq \frac{\sigma_\varphi}{B_\varphi}\frac{\partial\bar{v}_{||i}}{\partial x}\frac{\partial\tilde{\phi}}{\partial y} \implies \langle\Gamma_{v,x}^{TURB}\rangle_t \sim \left\langle\left(\frac{\partial\tilde{\phi}}{\partial y}\right)^2\right\rangle_t$$

- Using the pressure continuity equation:

$$\frac{\partial\tilde{p}}{\partial t} \sim \frac{\sigma_\varphi}{B_\varphi}\frac{\partial\tilde{\phi}}{\partial y}\frac{\partial\bar{p}}{\partial x} \underbrace{\implies}_{\partial_x\tilde{p}\sim\partial_x\bar{p}} \frac{\sigma_\varphi}{B_\varphi}\frac{\partial\tilde{\phi}}{\partial y} \sim \frac{\gamma}{k_x}$$

where $k_x = \sqrt{k_y/L_p}$ and $\gamma = c_s\sqrt{2/RL_p}$ [4].

- The turbulent radial momentum flux is then

$$\Gamma_x^{TURB} \simeq -B_\varphi\sqrt{\frac{2L_p}{R}}\frac{c_s}{k_y}\frac{\partial\bar{v}_{||i}}{\partial x}$$

2D EQUATION FOR THE EQUILIBRIUM FLOW

- We can write a 2D differential equation for the equilibrium parallel ion flow $\bar{v}_{||i}(x,y)$:

$$\boxed{\underbrace{-D_l\frac{\partial^2\bar{v}_{||i}}{\partial x^2} + v_l\frac{\partial\bar{v}_{||i}}{\partial x}}_{radial} + \underbrace{\frac{\sigma_\varphi}{|B_\varphi|}\frac{\partial\bar{\phi}}{\partial x}\frac{\partial\bar{v}_{||i}}{\partial y}}_{poloidal} + \underbrace{\alpha\sigma_y\bar{v}_{||i}\frac{\partial\bar{v}_{||i}}{\partial y}}_{parallel} + \underbrace{\frac{\alpha\sigma_y}{m_i\bar{n}}\frac{\partial\bar{p}}{\partial y}}_{generation} = 0}$$

where $D_l = \sqrt{\frac{2L_p}{R}}\frac{c_s}{k_y}$ and $v_l = D_l/2L_T$.

- The solution of this equation requires boundary conditions. At the magnetic presheath entrance [5],

$$\bar{v}_{||i}^{\pm} = \pm\sigma_y c_s^{\pm} - \frac{\sigma_y\sigma_\varphi}{\alpha|B_\varphi|}\left(\left.\frac{\partial\phi}{\partial x}\right|^{\pm} + \left.\frac{1}{en}\frac{\partial p_i}{\partial x}\right|^{\pm}\right)$$

ANALYTICAL SOLUTION FOR THE TOROIDAL ROTATION PROFILE

- Taylor expand the equilibrium profiles in y, and impose boundary conditions
- Assume we know $\delta n = (n^+ - n^-)/n_0$ and same for temperature
- Take $T_i \sim T_e$, $\phi \sim \Lambda T_e$, and $L_\phi \sim L_T$
- Consider $M = \sigma_\varphi\sigma_y\bar{v}_{||i}/c_s$ as the toroidal Mach number and assume $M(0,0) = 0$

$$\boxed{\begin{aligned} M(x,y) = &\left(\frac{\Lambda}{2\alpha}\frac{\rho_s}{L_T}e^{-x/L_T} - \sigma_\varphi\frac{\delta n + \delta T}{2}\right)\left(1 - e^{-x/L_T}\right) \\ &+ \sqrt{2}\sigma_y\sigma_\varphi\frac{y}{L_y} \\ &+ \left(\frac{2\rho_s}{\alpha L_T}e^{-x/L_T}\left(1 + \Lambda e^{-x/L_T}\right) + \sigma_\varphi\frac{\delta n + \delta T}{2}\left(1 - e^{-x/L_T}\right)\right)\frac{y^2}{L_y^2} \end{aligned}}$$

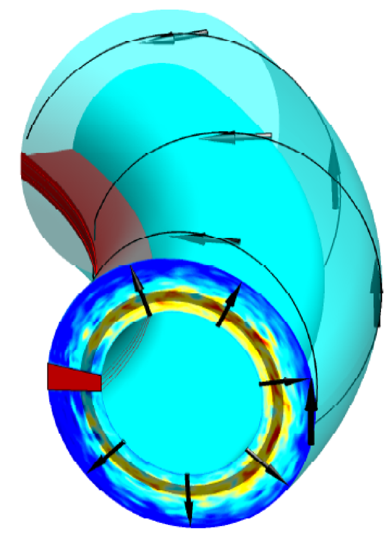
3. Global 3D simulations

MOTIVATION

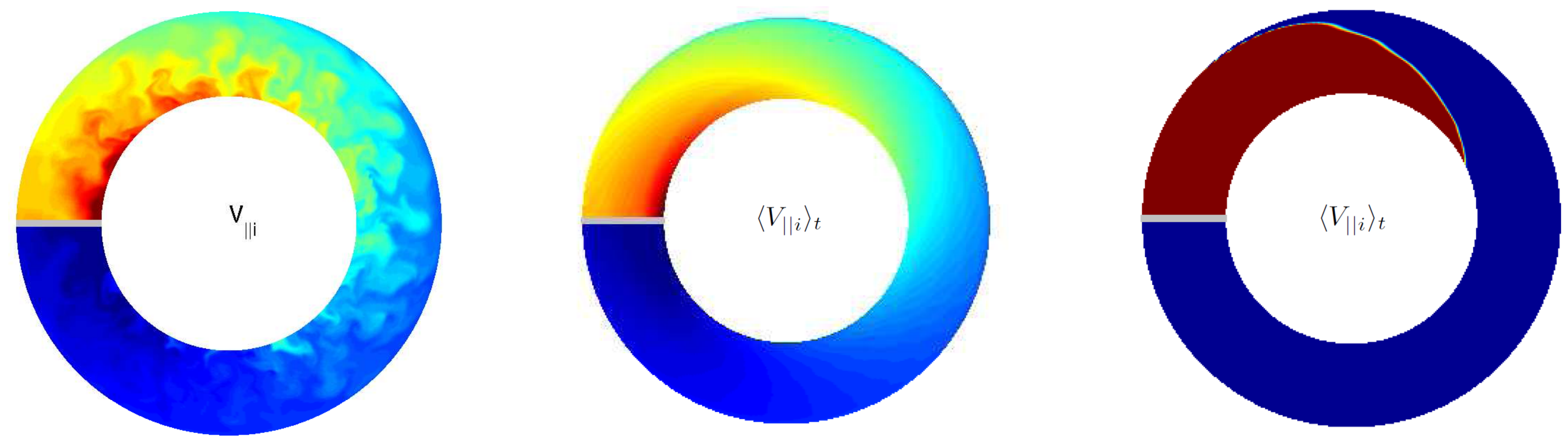
- Test rotation theory with 3D fluid simulations of SOL plasma turbulence in a simple configuration.

THE GBS CODE [6,7]

- Drift-reduced Braginskii equations.
- Evolves 3D fields: n, T_e, ϕ, $V_{||e}$, $V_{||i}$.
- No separation between equilibrium and fluctuations.
- Interplay between plasma outflow from the core, turbulent transport, and parallel losses [5].
- Circular concentric magnetic surfaces
- Radially localized n and T_e sources
- Toroidal limiter



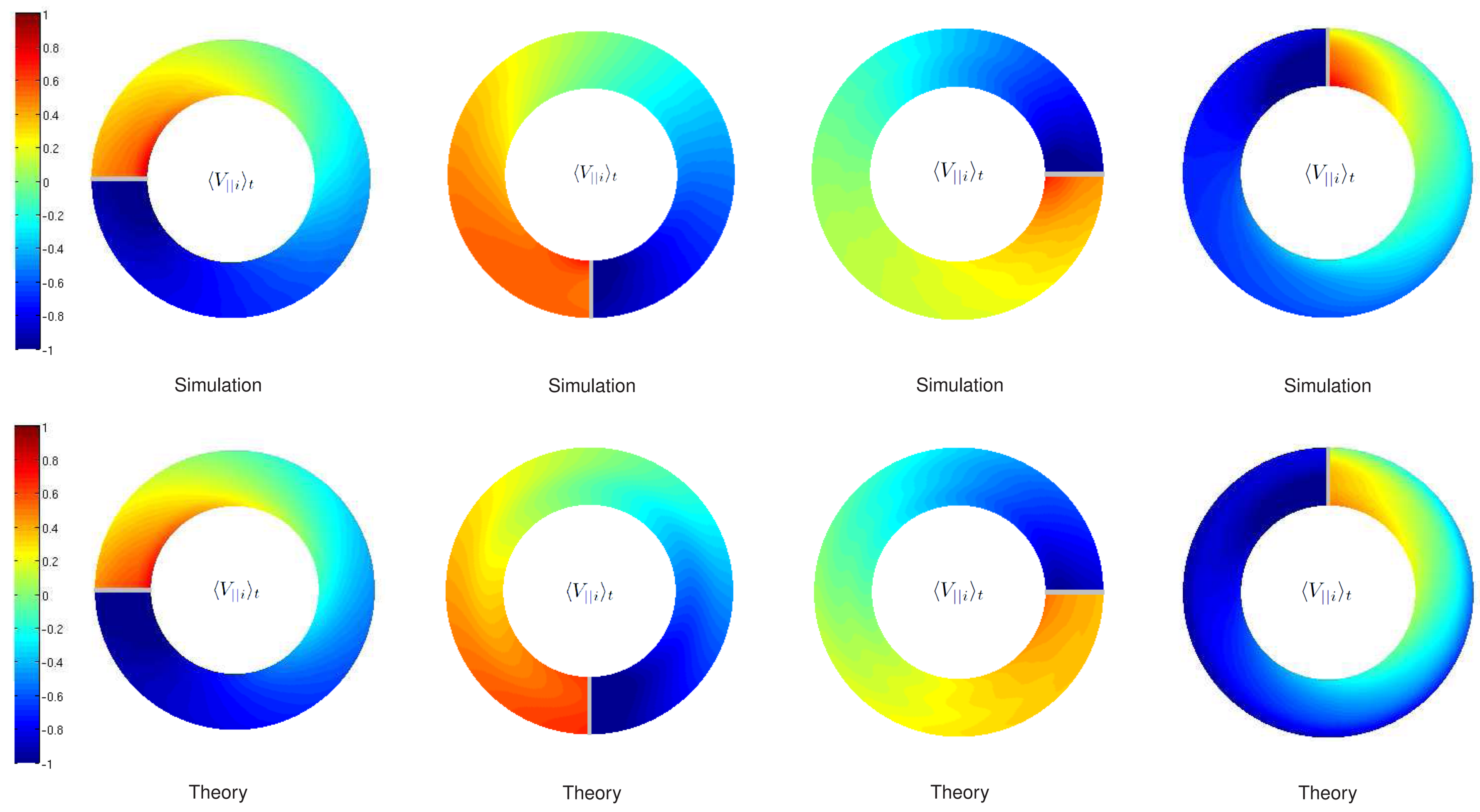
TOROIDAL ROTATION IN GBS SIMULATIONS



Snapshot | Time-average | Time-average +/-

There is a finite volume-averaged toroidal rotation

SIMULATION VS THEORY COMPARISON (different limiter positions)

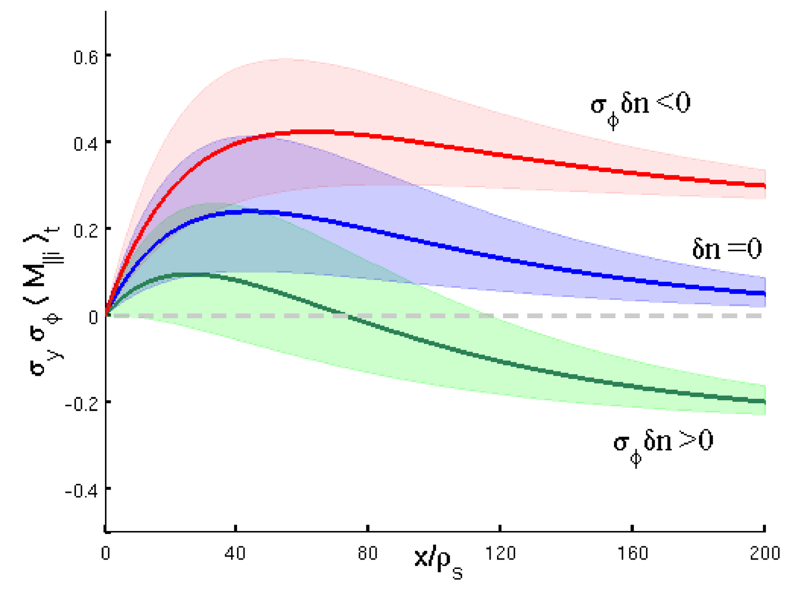


Good agreement between simulations and theory

4. Expected experimental trends

- Toroidal rotation profile, half way from the two divertor legs or limiter sides:

$$M(x,0) = \left(\underbrace{\frac{\Lambda}{2\alpha}\frac{\rho_s}{L_T}e^{-x/L_T}}_{sheath} - \underbrace{\frac{\sigma_\varphi}{2}\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right)}_{asymmetry}\right)\left(1 - e^{-x/L_T}\right)$$



- $|M_{||}| \lesssim 1$
- Typically co-current
- Rice scaling $V_\varphi \sim T_e/I_p$
- Can become counter-current by reversing **B** (σ_φ) or divertor position (δn)

Analytical trends agree with main observed experimental trends

5. References

[1] B. LaBombard *et al.*, Nucl. Fusion 52, 045010 (2004)
[2] B. LaBombard*et al.*, Phys. Plasmas 15, 056106 (2008)
[3] A. Zeiler *et al.*, Phys. Plasmas 4, 2134 (1997)
[4] P. Ricci and B. N. Rogers, Phys. Plasmas 16, 062303 (2009)
[5] J. Loizu *et al.*, Phys. Plasmas 19, 122307 (2012)
[6] P. Ricci *et al.*, Plasma Phys. Control. Fusion 54, 124047 (2012)
[7] P. Ricci *et al.*, Phys. Plasmas 18, 032109 (2011)